THE MATHEMATICS OF JUGGLING

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Mathematics and juggling both have long histories but on the surface seem to be totally disconnected disciplines. Juggling is the art of keeping multiple objects moving in interesting patterns. A juggler who is well practiced in this art can achieve the basic goal of keeping three objects airborne by utilizing a wide variety of patterns. In the last half a century mathematically minded jugglers came up with a system which formalizes the notion of a juggling pattern. This was useful at first because it was a convenient way for jugglers to tell each other what they were doing using precise language. The system is called siteswap and can be studied from a mathematical point of view. There have even been papers [1, 2, 3, 4, 7], Ph.D. Thesis [6] and a book [5] written on this subject. The type of mathematics involved is of a group theoretic/combinatorial flavor. In this project you will initially study the basic properties of juggling patterns. Once this is accomplished, you will be encouraged to formulate and attempt to prove, your own conjectures about the properties of these patterns. There are some surprising connections with other areas of mathematics which can also be investigated depending on your own motivation.

Prerequisites: This project will be open to anyone who has taken an introductory course in group theory.

Mentors: This project will be mentored in English by Oliver Sargent.

References

[1] Steve Butler and Ron Graham. Enumerating (multiplex) juggling sequences. *Ann. Comb.*, 13(4):413–424, 2010.

[2] Fan Chung and Ron Graham. Primitive juggling sequences. *Amer. Math. Monthly*, 115(3):185–194, 2008.

[3] Satyan L. Devadoss and John Mugno. Juggling braids and links. *Math. Intelligencer*, 29(3):15–22, 2007.

[4] Carsten Elsner, Dominic Klyve, and Erik R. Tou. A zeta function for juggling sequences. *J. Comb. Number Theory*, 4(1):53–65, 2012.

[5] Burkard Polster. *The mathematics of juggling*. Springer-Verlag, New York, 2003.

[6] Jonathan Derek Stadler. *Schur functions, juggling, and statistics on shuffled permutations*. ProQuest LLC, Ann Arbor, MI, 1997. Thesis (Ph.D.)–The Ohio State University.

[7] Yeung Yam and Jingyan Song. Extending Shannon's theorem to a general juggling pattern. *Stud. Appl. Math.*, 100(1):53–66, 1998.