Hilbert function spaces of analytic functions in a complex variable

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A Hilbert function space is a Hilbert space consisting of "bona fide" functions on a set X, in which the functional $f \mapsto f(x)$ that takes a function and returns its value at a point $x \in X$ is bounded. Here is an example: the space of all analytic functions $f(z) = \sum a_n z^n$ in the unit disc with square summable Taylor coefficients, i.e., $\sum |a_n|^2 < \infty$ (on the other hand $L^2(0,1)$ is **not** an example, because functions are only defined "almost everywhere").

The fact that point evaluation is a bounded functional makes a connection between function theory and Hilbert space theory, and a very rich theory emerges in which function theory and operator theory can benefit one from the other.

In this project you will learn the rudiments of Hilbert function space theory, and dive into the study of some concrete problems concerning Hilbert function spaces of recent interest. A central question that we wish to tackle is when these Hilbert function spaces are isomorphic.

Wait a minute: aren't all (separable) Hilbert spaces isomorphic? As abstract Hilbert spaces yes, but if you add the additional structure of a Hilbert function space things are quite different.

Requisites: A course in Functional Analysis, a course in Analytic Function Theory (or complex analysis).

Links:
https://noncommutativeanalysis.wordpress.com/2013/01/19/advanced-analysis-notes-16-hilbert-function-spaces-basics/

https://noncommutativeanalysis.wordpress.com/2013/01/23/advanced-analysis-notes-17-hilbert-function-spaces-picks-interpolation-theorem/