

Title: The Ryser Conjecture
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A Latin square is an $n \times n$ square filled by numbers from 1 to n , in such a way that every number appears precisely once in each row and column. Here is for example a 3×3 Latin square.

1	2	3
3	1	2
2	3	1

A transversal in a Latin square is a permutation submatrix containing n different numbers. Here is for example a transversal in the above Latin square.

1	2	3
3	1	2
2	3	1

A conjecture of Herbert Ryser from 1970 is that every odd order Latin square has a transversal.

This conjecture may look innocent, but it connects with deep problems in combinatorics. It is likely that its solution will lead to general insights. As of today, it is very far from being solved. An intriguing generalization of it was conjectured by Stein: an $n \times n$ square filled by numbers from 1 to n , so that every number appears n times, has an "almost transversal", namely a collection of $n-1$ entries, in different rows and columns and with different symbols.

The general field to which these conjectures belong is "matching theory". A matching is just a collection of disjoint sets, and given a collection of sets we usually want to find in it a large matching.

In the project we shall try to solve special cases, and examine whether there are many examples showing sharpness of the conjectures. For example – is there more than one canonical example in the odd case?